MATH 1070Q: Mathematics for Business and Economics

Lecture Slides

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Topic 9: Introduction to Probability with Permutations and Combinations

Reminders and To-Do

- Topic 9 will cover sections 7.6 in the text. In studying topic 9, we will also be reviewing and applying techniques from topic 8.
- Worksheet 9 at the end of class.

What is Probability?

One reason we have spent so much time learning to count is to study *probability*. Let's look at a toy example:

Example:

A fair coin is tossed once. The probability that the outcome is heads is 0.5, or 50%.

This is intuitively true, but how do we obtain the number 0.5 mathematically?

Choosing a subset of the sample space

The *sample space* for tossing a coin is $\{H, T\}$. The outcome of heads can be represented by the subset $\{H\}$. Note that then

$$\frac{|\{H\}|}{|\{H, T\}|} = \frac{1}{2} = 0.5.$$

Random Events

In other words: the probability of an outcome is the ratio of the number of times the outcome appears in the sample space to the entire sample space.

Definition

Let S be a sample space of some scenario.

1. A random event (or outcome) of that scenario is a subset $E \subseteq S$.
2. The probability $P(E)$ that the random event E will occur is

$$P(E) = \frac{|E|}{|S|}$$

Example of Random Events

A sample space for rolling a 6-sided die is the set $\{1, 2, 3, 4, 5, 6\}$. For each of the following, write down the corresponding random event and compute its probability:

1. Rolling a 2.
2. Rolling an even number.
3. Rolling a number bigger than 4.
4. Rolling a negative number.

Using Counting Techniques in Probability

As you can imagine, most often we will instead need to use counting techniques to count the sample space S and the random event E.

The rest of today will be extra practice with counting, but in the context of probability. We will study probability in more depth later.

Example

A fair coin is flipped 6 times. What is the probability of getting 4 heads?

Thought process: Since we are calculating $P(E) = \dfrac{|E|}{|S|}$, we need to...

1. Count the sample space S (the total number of possible 6-coin-flips.)
2. Count the random event E (the total number of ways to get 4 heads).

Example: Probability with Committees

Example: A 6-person research committee is to be randomly formed from a group consisting of 5 physicists and 7 mathematicians. Assuming every person has an equal probability of being selected, what is the probabilty the research committee contains an equal number of physicists and mathematicians?

Thought Process

Once again we need to count the sample space (all possibilities) and then count the random event. The random event needs to include outcomes in which there are an *equal number* of mathematicians and physicists.

Your Turn:

Example: A 6-person research committee is to be randomly formed from a group consisting of 5 physicists and 7 mathematicians. Assuming every person has an equal probability of being selected, what is the probabilty the research committee contains exactly 4 mathematicians?

Example: Probability of Organic Foods being Together

Example: A store is randomly arranging 8 different cereal brands on a shelf. There are 3 organic cereals and 5 non-organic cereals.

1. What is the probability that the organic cereals end up together and the non-organic cereals end up together?
2. What is the probability that just the organic cereals end up together?

Your Turn:

Example: A store is randomly arranging 8 different cereal brands on a shelf. There are 3 organic cereals and 5 non-organic cereals. What is the probability that just the non-organic cereals end up together?

Example: License Plates

Example: A standard CT license plate is a 7-character string, where the first two are letters and the last five are numbers. Assume all characters have an equal probability of being chosen. What is the probability a license plate will have no duplicate characters?

Your Turn:

A standard CT license plate is a 7-character string, where the first two are letters and the last five are numbers. Assume all characters have an equal probability of being chosen. What is the probability a license plate will have just no duplicate *numbers*?

Example: Defective Phones

Example: In a shipment of 20 smartphones, it turns out 5 are defective. A quality inspector randomly chooses 10 of the smartphones to test.

1. What is the probability that all 5 defective phones are chosen?
2. What is the probability that at least one defective phone is chosen?

Your Turn:

Example: In a shipment of 20 smartphones, it turns out 5 are defective. A quality inspector randomly chooses 10 of the smartphones to test. What is the probability that 3 defective phones are chosen?