MATH 1070Q: Mathematics for Business and Economics

Lecture Slides

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Topic 8: Permutations and Combinations

Reminders and To-Do

- Quiz (Topic 7) at beginning of class.
- Topic 8 will cover sections 7.1 & 7.4 in the text.
- Worksheet 8 at the end of class.

SC Exam Monday

Remember there is a second chance exam on Monday. Topic 8 will be tested for the first time, whereas the *second chance* portion is topics 5-7.

Midterm Review

Midterm reviews are due next Tuesday (14 July). I'm not sure what that means for you yet.

Counting Principle Revisited

Recall:

Counting Principle

The total number of outcomes for a sequence of k events, having respectively $n_1, n_2, \ldots, n_k$ many outcomes, is $n_1 \cdots n_2 \cdots n_k$.

With problems involving counting *without* replacement, we often encounter numbers of the form $7 \cdot 6 \cdot 5 \cdot 4$:

Example

If you own 7 mugs, how many ways can you give 4 friends your mugs?

The Factorial

Because the multiplication like $7 \cdot 6 \cdot 5 \cdot 4$, it is convenient to introduce a new operation:

Definition

For each $n \in \mathbb{N}$, we define the factorial of n to be the number:

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

If $n = 0$, we use the convention $0! = 1$.

Examples

Find the following factorials:

1. $4!$
2. $6!$

What do Factorials Count?

Example

How many ways can 5 distinct objects be arranged in a row?

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Factorials Count Arrangements

The number of ways to arrange n distinct objects is $n!$.

Techniques with Factorials

We can “pull apart” factorials:

Examples

1. Calculate $\dfrac{8!}{5!}$.
2. Calculate $\dfrac{10!}{6!}$.

We can rewrite our answer to the mug example using factorials:

Example

If you own 7 mugs, how many ways can you give 4 friends your mugs?

Permutations

In general, we have been studying permutations.

Definition

A permutation of distinct objects is an arrangement of those objects.

Formula

The number of ways to permute k objects from a pool of n objects is

$$P(n,k) = \frac{k!}{(n-k)!}$$

Examples of Permutations

Recall that $P(n,k) = \dfrac{k!}{(n-k)!}$.

Example 1

How many ways to arrange n distinct objects?

Example 2

Given 7 mugs, how many ways are there to arrange any 4 of them?

Remark

Sometimes you are not explicitly told a problem involves an arrangement (like the previous example), yet it is still a permutation problem.

Question

In the Olympics, 206 countries compete for the gold, silver, and bronze medals. How many different ways can the winners be chosen?

(A) $206!$

(B) $\dfrac{206!}{3}$

(C) $\dfrac{206!}{3!}$

(D) $206 \cdot 205 \cdot 204.$

More Examples of Permutations

Example 1

A history class is doing presentations and there are 10 groups in total.

1. How many orders can the groups do their presentations in?
2. Suppose now that 4 groups present on ancient history and 6 groups present on modern history. How many orders can the groups do their presentations in, given that all of the groups that did the same topic have to present in a row?

Example 2

A group of 8 friends lines up for a photo. How many ways can they arrange themselves if two specific friends, Bob and Alice, refuse to stand next to each other?

Combinations

Everything so far has implicitly required an ordering of objects: lining up, first-second-third places, order of presentations . . .

But sometimes we have scenarios in which order does *not* matter.

Example

Given 7 mugs, how many ways are there to donate any 4 of them?

Definition

A combination is any collection of objects (where order does *not* matter).

Solving a Combination Problem

Example

Given 7 mugs, how many ways are there to donate any 4 of them?

The answer is not $P(7,4) = \dfrac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4$, since it does not matter what order I donate the 4 mugs.

Since we want to count all those arrangements of 4 mugs once and there are 4! many arrangements, we must divide this out to get:

$$\frac{7 \cdot 6 \cdot 5 \cdot 4}{4!}$$

Combinations Formula

Formula: Number of Combinations

The number of combinations of k objects from a pool of n objects is

$$C(n, k) = \frac{n!}{k!(n-k)!}$$

Notice this is equivalent to $C(n, k) = \dfrac{P(n, k)}{k!}$, which was what we argued in the previous slide.

Example

How many ways can you choose 3 toppings for a pizza if there are 12 toppings to choose from?

Permutations vs. Combinations

One difficulty in solving combinatorics problems is figuring out if it is a *permutation* or *combination* problem.

Tips: Think of the following questions:

- Is the order an important factor?
- Are there any distinctions being made for each choice?
- Would the situation be different if the choices were the same, but rearranged?

Permutations generally answer *yes* to these questions, whereas combintions generally answer *no*.

Question

An award show is giving out 5 identical trophies to a group of 20 nominees. How many ways can they distribute the awards?

(A) $\dfrac{20!}{15!}$.

(B) $\dfrac{20!}{5! \cdot 15!}$.

(C) $\dfrac{5!}{20!}$.

(D) $\dfrac{5!}{15! \cdot 20!}$.

Drawing from Multiple Pools

Sometimes, you need to count something drawn multiple different pools.

Examples

Suppose a 6-person committee is going to be made from a group of 5 economists and 7 marketers.

1. How many ways can this committee be constructed under no restrictions?
2. How many ways if we require an equal number of economists and marketers?