MATH 1070Q: Mathematics for Business and Economics

Lecture Slides

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Topic 6: Set Operations

Reminders and To-Do

- Quiz (Topic 5) at beginning of class.
- Topic 6 will cover sections 1.4-1.5 in the text.
- Worksheet 6 at the end of class.
- Quiz (Topic 6) tomorrow morning.

Set Operations

Given two sets A and B, we have two natural ways to create a new set:

1. Putting all the elements of A and B into one big set $A \cup B$.
2. Putting only the elements in *both* A and B into one small set $A \cap B$.

Example

Let $A = \{1, x, 5, 7\}$ and $B = \{x, y, 1, 3, 2\}$. Find $A \cup B$ and $A \cap B$.

Union and Intersection

Definition

1. The union of sets A and B is the set $A \cup B$ consisting of elements from A or B.
2. The intersection of sets A and B is the set $A \cap B$ consisting of elements from *both* A and B.

Remark

Some people read $A \cup B$ as "A or B," and $A \cap B$ as "A and B."

Remark

If you have taken an intro logic course, you may have seen symbols $\vee$ and $\wedge$ for "and" and "or." The set complement operation corresponds to the logical "not" operation.

More Examples of Union and Intersection

Example

Let $A = \{1, 2, 3\}$ and $B = \{0, 3, 5\}$. Find $A \cup B$ and $A \cap B$.

Union is always a *larger* set that contains both A and B, whereas the intersection is always a *smaller* set that is contained in A and is contained in B. In symbols:

Subset Properties of Union and Intersection

- $A \subseteq A \cup B$ and $B \subseteq A \cup B$.
- $A \cap B \subseteq A$ and $A \cap B \subseteq B$.

Remark

Since $A \cap B \subseteq A$ and $A \subseteq A \cup B$, we always have $A \cap B \subseteq A \cup B$ also.

Question

Let S be the set of all sandwiches and C the set of all meals containing chicken. Which of the following does the set $S \cap C$ represent?

(A) The set of all meals.
(B) The set of all chicken sandwiches.
(C) The set of all sandwiches or meals with chicken.
(D) The set of chicken meals that are not sandwiches.

More Examples of Union and Intersection

Example 1

Let $A = \{2k \mid k \in \mathbb{Z}\}$ and $B = \{3k \mid k \in \mathbb{Z}\}$. What are the elements of the set $A \cap B$?

Example 2

Let $A = \{x \in \mathbb{R} \mid x < 0\}$ and $B = \{x \in \mathbb{R} \mid x > 0\}$. Find $A \cup B$ and $A \cap B$.

Union and Intersection of Three or More Sets

Question

Let A, B, and C be sets. Consider the following two sets:

- $(A \cup B) \cup C$.
- $A \cup (B \cup C)$.

Are these sets the same?

Example

Let's try the question with an explicit example: $A = \{1, 2, 3\}$, $B = \{2, 5, 7\}$, and $C = \{0, 2, 8\}$.

Union and Interesection of Three or More Sets

A similar result holds for the intersection operation:

Associativity of Union/Intersection

The union/intersection operations are associative, that is,

$$A \cup (B \cup C) = (A \cup B) \cup C.$$

We will now always unambiguously write

$$A \cup B \cup C$$

without the use of parenthesis.

Elements of multi-unions/intersections

- The set $A \cup B \cup C$ consists of elements in A or B or C.
- The set $A \cap B \cap C$ consists of elements in all three A and B and C.

Commutativity of Union/Intersection

Commutativity of Union/Intersection

The union/intersection operations are commutative, that is,

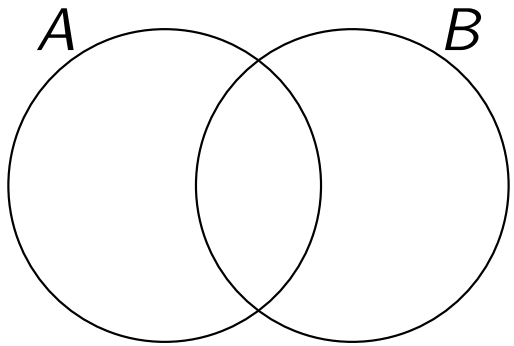
$$A \cup B = B \cup A \qquad \text{and } A \cap B = B \cap A.$$

Applying this to unions/intersections of more than two sets, we can rearrange however we'd like:

$$A \cup B \cup C = B \cup A \cup C = B \cup C \cup A = C \cup B \cup A = \cdots$$

Venn Diagrams

We can visually represent the union and intersection operations using Venn diagrams.



Question

The number of elements in the circle A is $|A|$, and similarly there are $|B|$ elements in B. Is it true that $|A \cup B| = |A| + |B|$?

Inclusion-Exclusion Principle

Since we are double-counting the overlap $A \cap B$, we have to subtract one of them.

Inclusion-Exclusion Principle

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

The inclusion-exclusion principle is the mathematical statement of a Venn diagram.

Question

Recall the inclusion-exclusion principle: $|A \cup B| = |A| + |B| - |A \cap B|$.

Question

A survey shows 50 students like jazz music, 20 students like vegetables, and 60 students like either jazz or rock. How many like both?

Application of Venn Diagrams

We can use Venn diagram to answer survey questions similar to the previous one.

Example

A chef surveyed 495 food enjoyers. The results are:

- 320 like fish.
- 395 like vegetables.
- 295 like beef.
- 280 like both fish and vegetables.
- 190 like both fish and beef.
- 245 like both vegetables and beef.
- 160 like all three.

- How many like exactly one food type?
- How many like exactly two food types?
- How many do not like vegetables?