MATH 1070Q: Mathematics for Business and Economics

Lecture Slides

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Topic 5: Basics of Sets

Reminders and To-Do

- Topic 5 will cover sections 1.1-1.3 of the text.
- Worksheet 5 at the end of class.
- Quiz (Topic 5) tomorrow morning.

Sets

Definition: Sets

A set is a collection of objects. The objects inside a set are called elements (or members) of the set.

Example:

The collection of all the students in this room forms a set.

Writing Sets

Most mathematicians use a capital letter to represent a set.

Example

Let C represent the set of all colors. Mathematically, one writes:

$$C = \{\text{red}, \text{blue}, \text{green}, \dots\}$$

- It does not matter what letter you choose. You could've chosen $A, B, Z, \Gamma, \Sigma, \dots$

To write if something is an element of a set, use the $\in$ symbol.

- Read "$\in$" as "...is in..."
- The color pink is in the set C, so $\text{pink} \in C$ in the previous example.

Subsets

Definition: Subset

Let A and B be two sets. If every element of A is also in B, we say that A is a subset of B. In this case we write $A \subseteq B$.

In symbols, we write: for every $x \in A$, we have $x \in B$.

Example:

Let $\mathbb{R}$ be the set of all numbers (the symbol will make sense later), and let $E = \{0, 2, 4, 6, \dots\}$ the set of all even natural numbers. Then $E \subseteq \mathbb{R}$.

Definition: Proper Subset

A set A is a proper subset of B if $A \subseteq B$ and B contains at least one element not in A.

Examples of Subsets

Example 1

The set $\{1, 2, 3\}$ is a subset of $\{0, 1, 2, 3, 4, 5\}$, written

$$\{1, 2, 3\} \subseteq \{0, 1, 2, 3, 4, 5\}.$$

It is also a proper subset, since there are elements (like $0, 4, 5$) in the second set not contained in the first set.

Example 2

The set $\{0, 1, 2, 3, 4, 5\}$ is a subset of $\{0, 1, 2, 3, 4, 5\}$. It is *not* a proper subset.

Question

If $A \subseteq B$ and $B \subseteq C$, is $A \subseteq C$?

Some Special Sets

Certain sets of numbers are used so often in mathematics that they get their own names:

Special Sets

- The set of natural numbers is $\mathbb{N} = \{0, 1, 2, 3, 4, 5, \dots\}$.
- The set of integers is $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$.
- The set of rational numbers is $\mathbb{Q} = \{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\}$.
- The set of real numbers is $\mathbb{R}$; it contains any non-imaginary number (all numbers for you guys).
- The empty set is $\varnothing = \{\ \}$. It contains nothing—literally empty.

Heirarchy of Numbers

1. $\{1, 2\} \subseteq \{0, 1, 2, 3, 4, 5\}$.
2. $\mathbb{N} \subseteq \mathbb{Z}$.
3. $\mathbb{Z} \subseteq \mathbb{Q}$.
4. $\mathbb{Q} \subseteq \mathbb{R}$.
5. Every set X has at least two subsets:
    - $\varnothing \subseteq X$.
    - What's the other one?

Question

Let E be the set of even natural numbers. Which of the following is FALSE?

A. $E = \{2, 4, 6, 8, 10, 12, 14, \dots\}$
B. $E \subseteq \mathbb{N}$
C. $17 \in E$
D. $\{2, 44, 198\} \subseteq E$

Writing Sets

Before, we described sets by simply listing out some of its elements. This is ambiguous, especially when one doesn't (or can't, like for an infinite set) list out *all* the elements of a set.

Set-Builder Notation

To describe a set, we write

$$\{x : c(x)\}$$

where x is a variable and $c(x)$ is a condition about x.

Example

The set of all colors is $C = \{x : x \text{ is a color}\}$. Here $c(x) =$ "x is a color."

More on Set-Builder Notation

Sometimes it is easier to first specify where x "lives", and then describe a condition on x.

Example 1

Let A be some set. The set $\{x \in A : c(x)\}$ contains all the elements "x in A such that $c(x)$ holds."

Example 2

Using set-builder notation, describe the set X of all even natural numbers.

Example 3

Using set-builder notation, describe the set Y of all even numbers.

More Examples using Set-Builder Notation

Example 1

Use set-builder notation to describe the set $\mathbb{Q}$ of all rational numbers.

Example 2

If $\mathbb{Z} = \left\{ \frac{p}{q} \in \mathbb{Q} : c\left(\frac{p}{q}\right) \right\}$, determine what the condition c is.

Note: the way of writing $\mathbb{Z}$ in example 2 explicitly shows that $\mathbb{Z} \subseteq \mathbb{Q}$.

Cardinality

We can ask how many elements a set has.

Definition

The cardinality of a set X is the number $|X|$ of elements in X, possibly infinity.

Example 1

If $X = \{a, b, c\}$, then $|X| = 3$.

Example 2

If $X = \mathbb{N} = \{0, 1, 2, 3, \ldots\}$, then $|X| = \infty$.

Question

True or False: If $A \subseteq B$, then $|A| \leq |B|$.

Complements

Let A and X be sets, with $A \subseteq X$. Then A divides X into two parts:

1. The elements in A.
2. The elements not in A.

Definition: Complement

The complement of a set A is the set of all elements in X that are not in A. We denote the complement as $X - A$. Some people use $A', \overline{A}, A^c$, or $X \setminus A$ instead.

Example:

If X is the set of all dishes on a menu, and $M \subseteq X$ is the subset of dishes containing meat, then what does $X - M$ represent?

Example of Complements

Let $X = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and

- $A = \{1, 2, 3, 4\}$,
- $B = \{2, 4, 6, 8\}$,
- $C = \{1, 8\}$.

What are $X - A$, $X - B$, and $X - C$?.

What is $X - X$? What is $X - \varnothing$?

More Examples on Complements

Example 1

The set $\mathbb{R} - \mathbb{Q}$ is called the set of irrational numbers. What is an example of an irrational number?

Example 2

Given that $\mathbb{Q} - \mathbb{Z} = \left\{ \dfrac{p}{q} \in \mathbb{Q} : c\left(\dfrac{p}{q}\right) \right\}$, determine the condition c.