MATH 1070Q: Mathematics for Business and Economics

Lecture Slides

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CAPS/SSS
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Topic 4: Compounding Interest and Methods of Savings/Investing

Reminders and To-Do

- Quiz (Topic 3) at the beginning of class.
- Topic 4 will cover sections 6.4 and 6.5 of the text.
- Worksheet 4 at the end of class.
- Quiz (Topic 4) will be part I of the SC exam on Monday.
- Holiday tomorrow (3 July) - no class.

Reminder: SC Exam Monday

Your first SC exam is next Monday and will cover topics 1-4. Continue to review previous topics this week to take advantage of the second chance. Some notes:

- Remember there are no "quiz" or "exam" grades. You are only graded on each topic.
- The grade for a given topic is the highest score between that topic's quiz and that topic's portion on the SC exam.

Simple vs. Compound Interest

In real life, most interest is compounding.

Definition

Interest that is based on both the original principal and previously earned interest is called compound interest.

In particular, compounding interest always affects the current balance, not just the principal.

Compounding Period

The amount of times you get an interest payment per year is called the compounding period.

Common Compounding Periods

- Annual: 1 time per year
- Semiannual: 2 times per year
- Quarterly: 4 times per year
- Monthly: 12 times per year
- Daily: 365 times per year

Compound Interest Formula

Formula: Compound Interest

The future/maturity value of an investment/loan with compounding interest is given by:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where. . .

- A: the new amount of money in the account.
- P: the original deposit (principal).
- r: the interest rate.
- n : number of times per year that interest is compounded.
- t: the time (in in years).

Reasoning for the Formula

Let's say a bank offers 6% interest, compounded monthly.

That 6% interest is NOT applied every time the interest is compounded. Instead, the bank "breaks up" that 6% into 12 pieces of 0.5%.

Each month, your bank takes your current balance and adds 0.5% of interest. The next month, they do the same thing with your balance at the end of the next month. They do this 12 times, and $12 \times 0.5 = 6$, giving you 6% "total" of interest.

So the $\frac{r}{n}$ comes from the fact that the interest is "broken up," and the nt comes from the fact that the interest is applied multiple times per year.

Example

You deposit \$500 into a bank account that earns 3% interest, compounded monthly. How much money is in your account after 2 years? What if the interest was compounded daily?

Effect of the Compounding Period

As the previous example implies, the more often the interest is compounded, the more money you will have at the end of your investment.

Effect of Compounding Period

More frequent compounding = more money earned.

Comparing Interest Rates and Compounding Periods

Recall that more frequent compounding gives more money. So if you are comparing two banks, one with a higher interest rate but less frequent compounding, and one with a lower interest rate but more frequent compounding, which bank is better?

Example

Bank A offers 12.7% interest compounded quarterly, and Bank B offers 12.6% interest compounded daily. Which bank offers a better deal?

Effective Interest Rate

The way we compare two banks with different interest rates and compounding periods is to calculate the effective interest rate for each.

The effective interest rate is the simple interest rate that would give you the same amount of money after 1 year of compound interest.

Definition

The effective interest rate is the simple interest rate that would give you the same amount of money after 1 year of compound interest:

$$E = \left(1 + \frac{r}{n}\right)^n - 1$$

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Present Value - Compounding Interest

Like with simple interest, we have a formula for the present value of an investment with compounding interest. The present value is the amount you need to deposit in order to have a certain amount of money at a future date.

Formula: Present Value

$$P = \frac{A}{\left(1 + \frac{r}{n}\right)^{nt}}$$

Example

You want \$40,000 saved up 10 years from now. If your bank offers 4% interest compounded monthly, how much must you deposit in order to save that amount?

Annuity

We have a technique to find present value, but in practice, one usually deposits a set amount of money every month.

Example

Alice wants to save \$27 000 in order to buy a car, in 3 years, without taking out a loan. She decides on a bank which offers 4.25% interest, compounded monthly. How much should she deposit every month in order to reach her goal?

We can calculate her present value to be \$23 774, but it is more practical to find how much she should deposit every month in order to reach her goal.

Monthly Deposit Formula

Formula: Monthly Deposit

The amount of money you need to deposit every month in order to reach a certain amount of money at a future date is given by:

$$d = \frac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt} - 1}$$

where...

- d: the monthly deposit.
- A: the future value of the account.
- r: the interest rate.
- n : number of times per year that interest is compounded.
- t: the time (in years).

Example

Example

Alice wants to save \$27 000 in order to buy a car, in 3 years, without taking out a loan. She decides on a bank which offers 4.25% interest, compounded monthly. How much should she deposit every month in order to reach her goal?

Formula: Monthly Deposit

$$d = \frac{A\left(\frac{r}{n}\right)}{\left(1+\frac{r}{n}\right)^{nt} - 1}$$

Future Value of Monthly Deposits

We can turn around the formula for monthly deposits to find the future value of an account with monthly deposits.

Formula: Future Value of Monthly Deposits

The future value of an account with monthly deposits d is given by:

$$A = d\,\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\left(\frac{r}{n}\right)}$$

Example

Bob saves \$250 every month in a bank account that earns 3.5% interest, compounded monthly.

1. How much money will he have in his account after 10 years?
2. How much money did he deposit in total?

Formula: Future Value of Monthly Deposits

$$A = d\,\frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\left(\frac{r}{n}\right)}$$