

MATH 1070Q: Mathematics for Business and Economics

Lecture Slides

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Topic 2: Ratios, Proportions, and Percents

Reminders and To-Do

- Quiz (Topic 1) at the beginning of class.
- Topic 2 will cover sections 5.4 and 6.1 of the text.
- Worksheet 2 will be done at the end of class.
- Quiz (Topic 2) Wednesday morning.

Rates

We use rates all the time in our lives. The speed you drive your car, the amount of money you earn per hour, the electricity consumed by a city each day, etc.

Definition: Rate

A **rate** is a comparison of one quantity in terms of another.

- We often write rates as a fraction.
- **Example:** What is your miles per gallon if you used 5 gallons to drive 125 miles?

Definition: Unit Rate

A **unit rate** is a rate with a denominator of 1.

- **Example:** 100 miles per hour (same 100 miles per “1” hour).
- **Example:** \$7 per pound.
- **Non-example:** \$14 per 2 pounds.

Example

One of the most common ways we use rates is when buying groceries.

Buying Apples

Suppose there are two bags of apples for sale:

- a 2 lb bag for \$4.
- a 5 lb bag for \$9.

Which is the better deal?

Example

You have two job offers. For Job 1, you work 40 hours a week and are paid \$1000 every week. For Job 2, you work 30 hours a week and are paid \$720 every week. Which job has the better hourly wage?

Choosing the Unit

When computing unit rates, you can choose what the unit will be.

Example:

A streaming service offers following plans: \$3 per day, \$20 per week, \$70 per month, \$860 per year. Which plan is the best (assuming you want streaming for the whole year)?

- Find the rate per year.
- Find the rate per day.
- Is our final answer the same regardless of what unit we chose?

Example

The dining hall offers meal plans at the following prices: \$12 per day, \$80 per week, \$340 per month, and \$4,000 per year. Which plan is the best, assuming you will eat at the dining hall every day?

Ratios

We often compare quantities that have the same units. For example: the amount of hours we spend sleeping every week—both of these quantities have the same units (units of time).

Definition: Ratio

A **ratio** is a comparison of two quantities that have the same units.

- Traditionally, ratios are written in three ways:
 - ▶ As a fraction: $\frac{a}{b}$.
 - ▶ As two numbers separated by a colon: $a : b$
 - ▶ As two numbers separated by the word “to”: a to b

Question: Is $a : b$ the same as $b : a$?

Computing Ratios

To compute a ratio a to b , you will divide the quantity a by the quantity b : put $\frac{a}{b}$, then put the fraction into simplest form.

Example:

The math department has 240 math majors and 25 faculty members.
Find:

- The ratio of math majors to faculty.
- The ratio of math majors to all members of the department.

Question

In a fish farm, the population of tilapia is 340 and the population of haddock is 140. What is the ratio of the total population of fish to the population of haddock?

- ① 7:17
- ② 17:7
- ③ 7:24
- ④ 24:7.

Proportions

When are ratios (or rates) equal? For example: the aspect ratio 3:4 is the same as 6:8. This is an example of a proportion.

Definition: Proportion

A **proportion** is an equation between two rates (or ratios):

$$\frac{a}{b} = \frac{c}{d}$$

Solving a Proportion

To solve a proportion, we use **cross multiplication**. That is we multiply:

$$\frac{a}{b} \times \frac{c}{d}$$

To get:

$$ad = bc.$$

When solving a proportion, you will be given 3 out of 4 of the variables and will need to find the last one.

Example:

Solve the proportion $\frac{7}{10} = \frac{x}{15}$.

Example

Solve the proportion $\frac{14}{x} = \frac{7}{3}$.

Example: Problem Solving using Ratios and Proportions

For some problems, you will combine ratios and proportions to solve them.

Example:

At UConn, there are 2 000 faculty members and 32 000 students. If UConn wants to increase its size to have 40 000 students, how many more faculty will they need to hire to keep the same student-faculty ratio?

- To solve this problem:
 - ➊ First, find the current (original) ratio of students to faculty.
 - ➋ Second, set up a proportion involving the above ratio and the new number of students & faculty.
 - ➌ Finally, find the number of faculty from above.

Example

A grocery store has 400 employees who serve 500 000 customers every month. If the store expands to serve 700 000 customers per month, how many more employees will the store need to hire in order to maintain the same customer-employee ratio?

Percents

Definition: Percent

A **percent** is a ratio with a denominator of 100.

Example: 50% is the ratio 50 : 100.

Converting between Decimals and Percents

① **Decimal to Percent:** multiply by 100.

$$0.57 \implies 0.57 \times 100 = 57 \implies 0.57 = 57\%$$

② **Percent to Decimal:** divide by 100:

$$57\% \implies \frac{57}{100} = 0.57 \implies 57\% = 0.57$$

Caution:

Percents can have decimals (23.76%), or be greater than 100 (137%).

Writing Percents as a Fraction

We sometimes want to write a percent as a fraction. For example,
 $25\% = \frac{1}{4}$.

To convert a general percent to a fraction, first remove the % sign, then divide by 100, and finally put the fraction in simplest terms.

Example:

Write 35% as a fraction.

Calculating Percents

Most of the time, we want to calculate some percent of some quantity. For example, when you calculate the tip on a restaurant bill, you are finding the percent of your subtotal.

Formula: Calculating a Percent

To find $x\%$ of a quantity y , we use the formula:

$$\frac{x}{100} \times y$$

Example:

What is 20% of 50?

The Basic Percent Equation

The formula on the previous slide is a nice shortcut for calculating $x\%$ of y . In general, we can use the following formula.

Formula: The Basic Percent Equation

Let p be the percent (written as a **decimal** or **fraction**), n_i be the initial number, and n_f be the final number. Then we have the following equation:

$$pn_i = n_f.$$

Examples:

- 1 What percent of 30 is 5?
- 2 If 28 is 45% of some number, what is the original number?

Example

A carpenter trims off some excess wood from a wooden block. Originally, the block weighed 500 grams. The block now weighs 475 grams after trimming. What percent of the block remained after trimming?

Example

40% of some number is 80. What is the original number?