MATH 1070Q: Mathematics for Business and Economics

Lecture Slides

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Topic 13: Basics of Functions

Reminders and To-Do

- Topic 13 will cover sections 7.10-7.11 in the text.
- Worksheet 13 at the end of class.

Functions

Often, it is not enough to just study sets. We are usually interested in how sets are related to each other. One way to witness this are functions.

Definition

Let A and B be sets.

- A function f from A to B, written $f : A \to B$, is a rule that associates to each element $x \in A$, a unique element $y \in B$. We write $f(x) = y$.
- The set A is called the domain of f.
- The set B is called the co-domain of f.

Note

This means that every element of the domain A is associated with *something* in B, and cannot be associated with more than 1 thing in B.

Examples and Non-Examples of Functions

Let $A = \{a, b\}$ and $B = \{0, 1\}$. Determine if the following defines a function $f : A \to B$:

1. $f(a) = 0$ and $f(b) = 1$.
2. $f(a) = 0$ and $f(b) = 0$.
3. $f(a) = 1$.
4. $f(a) = 0$, $f(b) = 0$, and $f(a) = 1$.

Representing Functions Diagrammatically

It is helpful to draw functions $f : A \to B$ as arrows from things in A to things in B. See board.

Image of a Function

As you may have noticed, given a function $f : A \to B$, it may be that f "misses" some elements of B.

Definition

Let $f : A \to B$ be a function. The image of f is the following subset of B:

$$\{y \in B \mid \text{there is an } x \in A \text{ such that } f(x) = y\}.$$

In other words, the image of f is all the stuff in B that f can "hit."

Examples

1. For each of the pictures on the board, determine the image of $f : A \to B$.
2. Let $f : \{a, b\} \to \{0, 1, 2\}$ be defined by $f(a) = 0$ and $f(b) = 2$. What is the image of f?
3. Let $f : \{a, b\} \to \{0, 1, 2\}$ be defined by $f(a) = 0$ and $f(b) = 0$. What is the image of f?
4. Let $f : \{a, b\} \to \{0, 1, 2\}$ be any function. Is it possible for there to be an f in which its image is *equal* to its co-domain?

Defining Functions by Equations

We have been defining functions by explicitly saying where it sends each element of its domain. What if we want to define a function $f : \mathbb{R} \to \mathbb{R}$?

Given $x \in \mathbb{R}$, we can describe $f(x)$ using equations.

Example

Write an equation $f(x) = \cdots$ for the function $f : \mathbb{R} \to \mathbb{R}$ which squares each input, for example $f(7) = 7^2 = 49$.

Examples and Non-Examples of Functions defined by Equations

The following are functions $f : \mathbb{R} \to \mathbb{R}$. Find $f(2)$ and $f(-1)$ for each:

1. $f(x) = 3x + 1$.
2. $f(x) = x^3 - 2x + 1$.

Note

Not every equation defines a function!

Explain why each of the following does *not* define a function $f : \mathbb{R} \to \mathbb{R}$.

1. $f(x) = \sqrt{x}$.
2. $f(x) = \dfrac{1}{x}$.

Domains of Definition

The equations $f(x) = \sqrt{x}$ and $f(x) = \frac{1}{x}$ do not define functions $\mathbb{R} \to \mathbb{R}$ because of domain issues. However, they do define functions if we restrict the domain:

1. The equation $f(x) = \sqrt{x}$ defines a function
$$f : \{x \in \mathbb{R} \mid x \geq 0\} \to \mathbb{R}.$$
2. The equation $f(x) = \dfrac{1}{x}$ defines a function
$$f : \{x \in \mathbb{R} \mid x \neq 0\} \to \mathbb{R}.$$

Definition

Given an equation $y = f(x)$, the largest set for which f becomes a function is called the domain of definition.

Examples of Domains of Definition

1. The domain of definition of $f(x) = \sqrt{x}$ is all the non-negative real numbers.
2. The domain of definition of $f(x) = \dfrac{1}{x}$ is all the non-zero real numbers.
3. What is the domain of definition of $f(x) = \dfrac{1}{\sqrt{x}}$?
4. What is the domain of definition of $f(x) = x^2$?

More Examples and Non-Examples of Functions

1. Is $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \sqrt{x}$ a function?
2. Is $f : \{x \in \mathbb{R} \mid x \geq 0\}$ given by $f(x) = \sqrt{x}$ a function?
3. Is $y = f(x)$ given by $y = x^2$ a function $\mathbb{R} \to \mathbb{R}$?
4. Is $y = f(x)$ given by $y^2 = x$ a function $\mathbb{R} \to \mathbb{R}$?
5. Is $y = f(x)$ given by $y - 3x = 2$ a function $\mathbb{R} \to \mathbb{R}$?

Image of Functions defined by Equations

1. What is the image of the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x$?
2. What is the image of the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$?
3. What is the image of the function $f : \{x \in \mathbb{R} \mid x \geq 0\} \to \mathbb{R}$ given by $f(x) = \sqrt{x}$?

Intervals

Sets like $\{x \in \mathbb{R} \mid x \geq 0\}$ are very common in analysis of functions, so we introduce some shorthand:

Notation

The interval $[a, b]$ is the following set of real numbers:

$$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}.$$

Similarly, we define (a, b) to be the set of all real numbers x such that $a < x < b$. We also use $[a, b)$ to mean $a \leq x < b$ and $(a, b]$ to mean $a < x \leq b$.

Example

The domain of definition of $f(x) = \sqrt{x}$ is $[0, \infty)$. The domain of definition of $f(x) = \dfrac{1}{x}$ is $(-\infty, 0) \cup (0, \infty)$.

More Examples

1. Let $f(x) = \dfrac{1}{\sqrt{x}}$. Write down the domain of definition using interval notation.
2. Let $f(x) = \dfrac{1}{x+5}$. Write down the domain of definition using interval notation.

Piecewise Functions

Sometimes more complicated functions need to be defined with multiple equations.

Example

Define a function $f : \mathbb{R} \to \mathbb{R}$ by

$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0. \end{cases}$$

This is an example of a piecewise function. Here, $f(x) = 1$ for any non-negative number x being inputed, and $f(x) = 0$ if a negative number x is inputed, e.g.,

$$f(1) = 1, f(5) = 1, f(0) = 1, \qquad f(-3) = 0, f(-27) = 0.$$

Example

Consider the following function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0. \end{cases}$$

Calculate $f(1)$, $f(5)$, $f(-7)$, and $f(-2)$. Does this function seem familiar to you?

Example

Consider the following function $f : \mathbb{Z} \to \mathbb{R}$ (note the domain!) given by

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is even} \\ x & \text{if } x \text{ is odd} \end{cases}$$

Calculate $f(1)$, $f(0)$, and $f(-2)$.