

MATH 1070Q: Mathematics for Business and Economics

Lecture Slides

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Topic 12: Binomial Distribution & Expectation

Reminders and To-Do

- Quiz (Topic 11) at the beginning of class.
- Topic 12 will cover sections 7.10-7.11 in the text.
- Worksheet 12 at the end of class.

SC Exam Monday

There is a **second chance exam** on Monday (20 July). Topic 12 will be tested for the first time, whereas topics 9-11 will comprise the “second chance” portion.

Binomial Experiments

Often, we are analyzing problems like the following:

Example

A coin is tossed 8 times. What is the probability of getting at least 5 heads?

In general, we will study:

Definition

A **binomial experiment** consists of multiple independent *trials* with only two outcomes, where the probabilities remain constant for each trial.

By **independent**, we mean that the outcome of one trial does not affect the other (like coin tosses).

Identifying Binomial Experiments

In the following, determine whether the experiment is binomial or not.

- ① Draw a card, note its suit, and replace it. Repeat this process 5 times.
- ② Draw 5 cards and count the number of ♣.
- ③ Draw a card, note whether it is a ♣ or not, and replace it. Repeat this process 5 times.

Towards Binomial Probability

Example 1

A 6-sided die is rolled 8 times. A roll will be considered “successful” if we roll a 1. It will be a “fail” if we roll anything else. Find:

- 1 The probability of getting exactly 2 successes.
- 2 The probability of getting exactly 3 successes.
- 3 The probability of getting exactly k successes for a number k between 0 and 8.

Formula for Binomial Probability

We have just found, as a special case, the following formula:

Formula

For a binomial experiment with n trials, the probability of k successes is

$$C(n, k) \cdot p^k \cdot (1 - p)^{n-k}.$$

Example

A coin is tossed 100 times. What is the probability of tossing exactly 50 heads? Why is the probability so low if $50/100 = 1/2$?

Applying Binomial Probability

Example

A coin is tossed 8 times. Let E be the event of tossing exactly 3 heads and F the event of tossing exactly 4 heads. Are E and F mutually exclusive?

Example

A coin is tossed 8 times. Find:

- 1 The probability of getting 2 or fewer heads.
- 2 The probability of getting at least 4 heads.

Example

Team A and team B are playing against each other in a “best-of” series. Find the probability that A will win the series against B in each of the following scenarios:

- ① In a best-of-5 series, where the probability A wins a single game against B is 0.51.
- ② In a best-of-3 series, where the probability A wins a single game against B is 0.60.

Playing the Lottery

Scratch-off lottery tickets often have various prizes, where each prize has its own probability of occurring.

Example

A ticket gives \$0 with a probability of 0.8, \$10 with a probability of 0.15, \$50 with a probability of 0.04, and \$200 with a probability of 0.01.

How much can you “expect” to win from this ticket? If the ticket costs \$10, is it a worthwhile investment?

95% of the time, you lose or break even. But maybe the small chance of winning big cancels this out.

Expectation

Definition

For an event X with multiple outcomes, the average of those outcomes in the long run is called the **expectation** of X . We write the expectation as $E[X]$.

Formula: Expectation

If an event X has the possible outcomes $S_1, S_2, \dots, S_k$, each with probability $P(S_1), P(S_2), \dots, P(S_k)$, the expectation of X is:

$$E[X] = S_1 \cdot P(S_1) + S_2 \cdot P(S_2) + \cdots + S_k \cdot P(S_k)$$

Example

Example: A \$10 lottery ticket gives \$0 with a probability of 0.8, \$10 with a probability of 0.15, \$50 with a probability of 0.04, and \$200 with a probability of 0.01. What is the expected value of playing the game?

Expectation and Complements

Example: Renter's Insurance

Suppose an insurance company offers a \$15 000 policy for \$35. The chances of a given apartment being broken into and robbed for the year are 0.007926. Will the company, on average, lose money or profit from selling these policies?

Knowing how to work with expectation can help price products. See the HW for a walk-through example.