MATH 1070Q: Mathematics for Business and Economics

Lecture Slides

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Topic 11: Mutual Exclusivity & Conditional Probability

Reminders and To-Do

- Quiz (Topic 10) at the beginning of class.
- Topic 11 will cover sections 7.8 & 7.9 in the text.
- Worksheet 11 at the end of class.

Review: Complement Probabilities

Last time, we introduced the notion of complement probabilities:

Definition

If E is a random event for some scenario with sample space S, then the $E' = S - E$ is the event in which E does *not* happen. Furthermore,

$$P(E') = 1 - P(E).$$

We have related one set operation (complementation) with probability. What about the other operations?

Review: Unions and Intersections

Recall that if A and B are sets, then their union $A \cup B$ is the set of elements in A or B:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

Alternatively, their intersection $A \cap B$ is the set of elements in A and B:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}.$$

Unions and Intersections of Random Events

Let E and F be two random events.

Definition

- The event in which E or F happens is $E \cup F$.
- The event in which E and F happens is $E \cap F$.

Example

A 6-sided die is rolled. Let E be the event of obtaining an even number. Let F be the event of obtaining a number bigger than 4.

1. Write down the sample space S and the two events E and F.
2. Write down the event in which E or F happens.
3. Write down the event in which both E and F happens.

Probability with Unions and Intersections

Recall the inclusion-exclusion principle: for any sets A and B,

$$|A \cup B| = |A| + |B| - |A \cap B|.$$

If we apply the inclusion-exclusion principle to calculate $P(E \cup F)$ for random events E and F, we obtain:

Formula: Probabilistic Inclusion-Exclusion Principle

For two random events E and F,

$$P(E \cup F) = P(E) + P(F) - P(E \cap F).$$

Example

Example

A 6-sided die is rolled. Let E be the event of obtaining an even number. Let F be the event of obtaining a number bigger than 4. We found:

- $S = \{1, 2, 3, 4, 5, 6\}$.
- $E = \{2, 4, 6\}$ and $F = \{5, 6\}$.
- $E \cup F = \{2, 4, 5, 6\}$ and $E \cap F = \{6\}$.

Use the inclusion-exclusion formula to calculate...

1. ...the probability of rolling an even number *or* a number bigger than 4.
2. ...the probability of rolling an even number *and* a number bigger than 4.

Compare with the direct method of calculating probabilities.

Example

You are about to draw a card at random from a deck containing only these 10 cards: A$\heartsuit$, A$\spadesuit$, A$\clubsuit$, A$\diamondsuit$, K$\spadesuit$, K$\clubsuit$, Q$\heartsuit$, Q$\spadesuit$, J$\heartsuit$, J$\spadesuit$.

1. You draw a $\spadesuit$ or a $\clubsuit$.
2. You draw an ace or a $\heartsuit$.

Mutually Exclusive Events

In the previous example, we saw that it may be possible that two events cannot happen at the same time.

Definition

Two random events E and F are said to be mutually exclusive if they cannot happen simultaneously. Mathematically,

$$E \cap F = \varnothing$$

In this case, since $P(E \cap F) = P(\varnothing) = 0$, we have:

Formula: Probability of Mutually Exclusive Events

If E and F are mutually exclusive, then

$$P(E \cup F) = P(E) + P(F).$$

Example

A coin is tossed 6 times. Let E be the event of obtaining 0 heads and F the event of obtaining 1 head.

1. Are E and F mutually exclusive?
2. What is the probability that E or F will happen?

Remember: we did a similar problem before, but phrased as "what is the probability of obtaining *at least* 2 heads." The only ways to *not* get at least 2 heads is to do either E or F.

Conditional Probability

Example

A coin is tossed twice.

1. What is the probability of getting all heads?
2. Now suppose the first toss lands on heads. Now, what is the probability of getting all heads?

Sometimes we need to analyze probability after certain conditions are met, which is called conditional probability.

Multi-stage Experiments

Often, like in the previous example, conditional probabilities are used in multi-stage experiments.

Definition

Let F be the first random event and E be the second random event for some multi-stage scenario. Then, the probability of E given F is denoted by

$$P(E \mid F)$$

and is the probability that E will occur assuming that F has *already occured*.

Example

Alice is playing a game with Bob in which she flips a coin 3 times. If the coin lands on heads more than tails, Alice wins; otherwise, Bob wins. Define the following random events:

- A is the event in which Alice wins.
- H is the event in which the *first* flip is heads.
- T is the event in which the *first* flip is tails.

Questions

1. Write out the sample space S for flipping a coin 3 times.
2. Write out the subsets A, H, and T of S.
3. Find $P(A)$, $P(A \mid H)$, and $P(A \mid T)$.

Formulas for Conditional Probability

If we are calculating $P(E \mid F)$, we can *restrict* the sample space S down to F. Then:

Formula: Conditional Probability

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)}$$

This formula implies also that:

Formula: Probability of E "and" F

$$P(E \cap F) = P(F) \cdot P(E \mid F)$$

Example

A school club has 10 members: 4 seniors, 3 juniors, 2 sophomores, and 1 first-year. Two members will be selected for president and vice-president.

1. What is the probability that a sophomore is selected for president and then a senior is selected for vice-president?
2. What is the probability that both roles are taken by seniors?