Homework: Topics 9 - 12

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Topic 9: Introduction to Probability with Permutations and Combinations

Problem 1. Do the following problems in section 7.6 of the text: 8-9, 11-15.

Problem 2. An 8-person research committee is to be formed from a group consisting of 13 mathematicians and 9 physicists.

(a) Assume all 8 people are selected randomly. What is the probability that all 8 are mathematicians? What is the probability that all 8 are physicists?

(b) What is the probability that the committee has an equal number of physicists and mathematicians?

Problem 3. A fair coin is tossed 10 times.

(a) How many different outcomes are possible?

(b) How many different outcomes are possible such that at least 7 coins come up as heads? (*Hint: to have at least 7 heads is to have exactly 7 heads, or exactly 8 heads, or... so on. Count how many ways to do each of these to find the total.*)

(c) What is the probability of getting at least 7 heads?

Problem 4. A group of 7 friends line up for a photo.

(a) How many different ways can these 7 friends line up in a row?

(b) Bob and Alice are part of the 7-friend group. Now, Bob seems to like Alice and is wondering what the probability of them standing next to each other is, assuming the 7 friends line up randomly. What is the probability Bob and Alice stand next to each other?

(c) Now, Bob has done something awkward and Alice does not want to stand next to him. Alice is wondering what the probability they *don't* stand next to each other is. What is it, assuming the 7 friends line up randomly?

Topic 10: General Probability & Odds

Problem 1. Do the following problems in section 7.5 of the text: 1-16.

Problem 2. For each of the following, use the fact that $P(E) = 1 - P(E')$, where E' is the event in which E does *not* happen.

(a) A 6-sided die is rolled. What is the probability of rolling a number bigger than 2?

(b) A 6-sided die is rolled *twice*. What is the probability that the sum rolled is 10 or less?

Problem 3. Do the following problems in section 7.7 of the text: 1-32 (odd numbered problems).

Problem 4. A bag contains a number of red and blue marbles. The odds of choosing a blue marble are 7:3. You draw from the bag 5 times.

(a) You draw from the bag **with** replacement. What is the probability of drawing a red marble at all, if there are 30 marbles? Is the number of marbles relevant?

(b) You draw from the bag **without** replacement. What is the probability of drawing a red marble at all, if there are 30 marbles? Is the number of marbles relevant?

Topic 11: Mutual Exclusivity & Conditional Probability

Problem 1. Do the following problems in section 7.8 of the text: 7-14

Problem 2. You are given a 3-letter code. What is the probability that there is a repeated letter?

Problem 3. Do the following problems in section 7.9 of the text: 7-16 (odd numbered problems), 31-34.

Problem 4 (The Birthday "Paradox"). Assume that leap years do not exist, so that the probability of being born on a given day is $\frac{1}{365}$.

(a) Let us start with a group of two people: Bob and Alice. What is the probability that Alice has a different birthday than Bob?

(b) Now suppose Joe joins the group. Given that Bob and Alice do not share the same birthday, what is the probability that Joe has a different birthday than Bob and Alice?

(c) Using the pattern established in parts (a) and (b), what is the probability that a group of 5 people do not share a birthday?

(d) What is the probability that a group of 23 people **do** share a birthday? The surprisingly high number you obtain here is why this problem is sometimes called the "birthday paradox."

Topic 12: Binomial Distributions & Expectation

Problem 1. Do the following problems in section 7.10 of the text: 1-12, 17-19.

Problem 2. In problem 2 of topic 9, you computed the probability of getting at least 7 heads in 10 coin tosses. Suppose now a coin is tossed 20 times.

(a) Use binomial distribution techniques to find the probability of getting at least 14 heads.

(b) Bob is confused why the probability you found in part (a) does not match the one you found in problem 2 of topic 9, even though $7/10 = 14/20$. Explain this phenomenon to Bob.

Problem 3. Do the following problems in section 7.11 of the text: 3-11.

Problem 4. Bob wants to start a car insurance company but has no idea how much his 1-year policies should sell for. Bob wishes he paid attention in MATH 1070Q, since he has to now pay you money to help him figure out a good selling price. Assume Bob wants to profit around $500 for each policy.

(a) Alice owns a $30 000 car. Suppose that Alice is a safe driver so that the probability of her car being totaled in a given year is 0.0187. How much should Bob sell a 1-year policy to Alice for? (*Hint: if Alice totals the car, Bob will need to pay $30 000. Bob needs to determine the sell price S so that the expectation value is equal to his profit of $500.*)

(b) How much should Bob charge Alice if Alice owns a $80 000 car?

(c) Suppose instead that Alice is a risky driver and has a 0.0831 probability of totaling her car in a given year. How much should Bob charge Alice, assuming Alice owns a $30 000 car?