Homework: Topics 5 - 8

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Topic 5: Basics of Sets

Problem 1. Do the following exercises from section 1.1 in the textbook: 9-12.

Problem 2. Do the following exercises from section 1.2 in the textbook: 1-2.

Problem 3. Do the following exercises from section 1.3 in the textbook: 10-11, 17, 25-27.

Problem 4. Answer the following true/false questions for arbitrary sets A, B, and C. If true, justify. If false, provide a counterexample (that is, an example that falsifies the statement).

(a) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

(b) If $A \subseteq B$, then $|A| \leq |B|$.

(c) If $A \subsetneq B$, then $|A| < |B|$.

Problem 5. For a set A, we define the **power set** of A to be the set $\mathcal{P}(A)$ of all subsets of A.

(a) Let $A = \{1, 2, 3\}$. Find $\mathcal{P}(A)$. (*Hint:* remember that $\varnothing$ and A are always subsets of A).

(b) Compute $|A|$ and $|\mathcal{P}(A)|$. Observe that $|A| \leq |\mathcal{P}(A)|$ here.

(c) Now let A be any arbitrary set. Explain why $|A| \leq |\mathcal{P}(A)|$ is guaranteed to hold. (*Hint*: if $x \in A$, then $\{x\} \in \mathcal{P}(A)$.)

Topic 6: Set Operations

Problem 1. Do the following exercises from section 1.4 in the textbook: 1-2, 5-6, 9, 12-14.

Problem 2. Do the following exercises from section 1.5 in the textbook: 47-48.

Problem 3. A music teacher surveyed 495 students. The results are:

- 320 students like rap music. • 395 students like rock music. •295 students like heavy metal music.
- 280 students like rap music and rock music. • 190 students like rap music and heavy metal music.
- 245 students like both rock music and heavy metal music. • 160 students like all three.

(a) Draw a Venn diagram representing the survey results.

(b) How many students liked exactly one type of music?

(c) How many students liked exactly two types of music?

(d) How many students did not like rock?

Problem 4. Use Venn diagrams to show the following statements (called **DeMorgan's laws**) are true:

(a) $U - (A \cup B) = (U - A) \cap (U - B)$.

(b) $U - (A \cap B) = (U - A) \cup (U - B)$.

Topic 7: Counting Techniques and the Sample Space

Problem 1. Do the following exercises from section 7.1 in the textbook: 1-2, 7.

Problem 2. Do the following exercises from section 7.4 in the textbook: 11-12, 22-23, 28.

Problem 3. The goal of this problem is to introduce the technique of *counting without replacement*. Recall that the English alphabet has 26 characters. We want to count how many 5-character strings are possible if we do *not* allow duplicate characters (e.g., the string AXBKH is possible, but ABKXB is not).

(a) A 5-letter string has the form "_ _ _ _ _", where each blank represents a letter from the alphabet. How many choices of letters are there for the first blank?

(b) Now suppose a choice for the first blank has been made, so our string looks for example like "A _ _ _ _". Given that duplicate letters are *not* allowed, how many choices of letters can be made for the second blank?

(c) Similarly, determine how many choices remain for the third, fourth, and fifth blanks.

(d) Apply the counting principle to count how many 5-letter strings are possible if we do not allow duplicate letters, finishing the problem.

Problem 4. Count the following:

(a) A standard CT license plate is a 7-character string, where the first two are letters and the last five are numbers. How many possible plates are there?

(b) How many plates are there if we do not allow duplicate characters?

Topic 8: Permutations and Combinations

Problem 1. Do the following problems from section 7.2 in the textbook: 3-5, 15-22.

Problem 2. Do the following problems from section 7.3 in the textbook: 22-24, 29-35.

Problem 3. How many ways can a store arrange 8 brands of cereal on a shelf if 3 brands are organic, 5 are non-organic, the organic brands are together, and the non-organic brands stay together?

Problem 4. In a shipment of 20 smartphones, 5 are defective. How many ways can a quality inspector randomly test 10 smartphones, of which 5 are defective?

Problem 5. A 6-person committee is to be chosen from a group of 5 physicists and 7 mathematicians.

(a) How many possible committees can be made under no restrictions?

(b) Suppose now the committee is to contain an equal number of physicists and mathematicians. How many possible committees of this type are possible?