Homework: Topics 13 - 16

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Topic 13: Basics of Functions

Problem 1. Do the following problems in section 5.7 of the text: 8-9, 12-13, 15-17.

Problem 2. For the following functions, determine its (maximal) domain of definition and its image.

(a) $f(x) = x^2$.

(b) $f(x) = \sqrt{x}$.

(c) $f(x) = \frac{1}{x}$.

(d) $f(x) = \begin{cases} \dfrac{n}{2}, & \text{if } x = 2n \text{ for some } n \in \mathbb{Z}. \\ 3n+1, & \text{if } x = 2n+1 \text{ for some } n \in \mathbb{Z}. \end{cases}$.

Problem 3 .

(a) **(From R. Hammack's *Book of Proof*)** There are four different functions $f : \{a, b\} \to \{0, 1\}$. List them all (you may use diagrams).

(b) How many functions $f : \{a, b, c, d\} \to \{0, 1\}$ are there? Explain how you know. (*Hint: each element of the domain can be associated to exactly one element in the codomain. Using counting techniques to count how many ways you can do this.*)

Problem 4. Recall that an equation is said to be *linear* if it is of the form $ax + b = 0$. In general, a function $f : \mathbb{R} \to \mathbb{R}$ is said to be *linear* if $f(a + b) = f(a) + f(b)$ and $f(ka) = kf(a)$ for numbers k, a, and b. We will study systems of linear equations later.

(a) Is the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = 3x$ linear? Explain.

(b) Is the function $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$ linear? Explain.

Topic 14: Slope and the Graph of a Function

Problem 1. Do the following problems in section 5.8 of the text: 1-32 (odd numbered problems).

Problem 2. Let $f(x) = x^2$.

(a) Fix any real number h. Compute and simplify $f(x+h)$.

(b) Find the slope of the line between the points $(x, f(x))$ and $(x+h, f(x+h))$. Your answer will involve both x and h. (*Remark: what you found here is sometimes called the "difference quotient." You will see more of this if you continue to MATH 1071Q.*)

Problem 3. Bob and Alice started receiving teddy bear's the day they were born. Bob was given 12 teddy bears when he was born, whereas Alice was given just 5. Each year, Bob is given another 2 bears, and Alice is given another 5.

(a) Find a function $f(t)$ that represents the number of Bob's bears after t years. Find a function $g(t)$ for Alice's bears.

(b) What are the y-intercepts for f and g?

(c) Graph the functions f and g. Do these functions intersect? If so, find the point of intersection and interpret your answer. If not, explain why not.

Topic 15: Systems of Linear Equations

Problem 1. Do the following problems in section 5.9 of the text: 5-27 (odd numbered problems).

Problem 2. The total number of calories in 2 hot dogs and 3 cups of cottage cheese is 960 calories. The total number of calories in 5 hot dogs and 2 cups of cottage cheese is 1,190 calories. How many calories are in a hot dog? How many calories are in a cup of cottage cheese?

Problem 3. Consider the following system of linear equations with unknown constants a and b:

$$\begin{cases} ax - 3y = 21 \\ bx - 3y = 11 \end{cases}$$

For what value(s) of $\frac{a}{b}$ does the above system admit *no* solutions?

Problem 4. The methods we learn generalize to systems of linear equations with more than two variables. Solve the following system of linear equations in three variables:

$$\begin{cases} 2x + 5y + 2x = -38 \\ 3x - 2y + 4z = 17 \\ -6x + y - 7z = -12 \end{cases}$$

Topic 16: Matrix Methods

Problem 1. Consider the following augmented matrix:

$$\left(\begin{array}{ccc|c} 1 & -6 & 2 & 0 \\ 2 & -8 & 10 & 4 \\ 3 & -4 & -1 & 2 \end{array}\right).$$

Perform the indicated elementary row operations:

(a) $\frac{1}{2}R_2$.

(b) Swap R_1 and R_3.

(c) Subtract $6R_3$ from R_1.

Problem 2. Consider the following system of equations:

$$\begin{cases} x - 7y = -11 \\ 5x + 2y = -18 \end{cases}$$

(a) Put the above system into matrix form $A\vec{x} = \vec{b}$.

(b) Solve the system by multiplying through with the inverse matrix A^{-1}.

Problem 3. Consider the following system of equations:

$$\begin{cases} 6x - 5y = 8 \\ -12x + 2y = 0 \end{cases}$$

Convert the system into an augmented matrix and solve by transforming the matrix into reduced row echelon form.

Problem 4. Determine if each of the following matrices are invertible or not. If applicable, find the inverse.

(a) $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

(b) $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.

(c) $A = \begin{pmatrix} 3 & 9 \\ -2 & -6 \end{pmatrix}$.